SMALL PERTURBATIONS OF UNSTEADY ONE-DIMENSIONAL AXISYMMETRIC MOTIONS OF AN IDEAL INCOMPRESSIBLE LIQUID

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The investigation of the stability of unsteady motions is combined with the study of the asymptotic representation of the solutions of the equations of small perturbations which satisfy given initial and boundary conditions. In most cases it is practically impossible to determine the asymptotic development of small perturbations because of the complexity of the equations describing the evolution of small perturbations. The solution of the problem under discussion is considerably simplified if the equations of small perturbations can be reduced to a system of ordinary differential equations. A number of examples of unsteady motions of an ideal incompressible liquid are known for which the asymptotic development of small perturbations has been investigated [1-5]. In most of the references cited the equations of small perturbations were reduced to a system of ordinary differential equations by assuming that the perturbed motion is irrotational. On the other hand, it is known [1, 4] that taking account of rotational perturbations can have a pronounced effect on conclusions about the stability of the motion under study. The present paper treats plane unsteady motions of an ideal incompressible liquid. It is shown that for a broad class of axisymmetric one-dimensional motions the equations of small perturbations can be reduced to a system of ordinary differential equations. No additional restrictions, such as the requirement that the perturbed motion be irrotational, are imposed. As an application of this result we have investigated the evolution of small perturbations of the motion of a rotating ring of an ideal incompressible liquid toward the center under the action of an external pressure.

1. Fundamental Motion. Axisymmetric one-dimensional motions of an ideal incompressible liquid are determined by the solutions of the following system of equations:

$$\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}-\frac{v_\varphi^2}{r}+\frac{1}{\rho_0}\frac{\partial p}{\partial r}=0, \tag{1.1}$$

$$\partial v_\varphi/\partial t+v_r\partial v_\varphi/\partial r+v_rv_\varphi/r=0,\ \partial v_r/\partial r+v_r/r=0,$$

where p is the pressure, ρ_0 is the density, and v_r and v_φ are the velocity components along the axes of the polar coordinate system, r, φ

For definiteness we assume that Eqs. (1.1) describe the motion of a liquid in the ring $R_1(t)\le r\le R_2(t)$ with free surfaces $r=R_i(t)$ $(i=1, 2)$. Since generally there are free surfaces in perturbed motions also, it is expedient to use the Lagrangian form of the hydrodynamics equations. We define the independent Lagrangian variables ρ and θ as the values of the polar coordinates r and φ of particles at time t=0.

If we introduce dimensionless dependent and independent variables by the expressions

$$\rho=R_{10}(\xi+1),\quad t=T\tau,\quad r=R_{10}s,\quad p=\frac{\rho_0R_{10}^2}{T^2}p', \tag{1.2}$$

where R_{10} is the initial inner radius of the ring and T is the characteristic time, the equations of plane parallel motions of an ideal incompressible liquid in these variables can be written in the form

$$\frac{\partial s}{\partial\xi}\left[\frac{\partial^2 s}{\partial\tau^2}-s\left(\frac{\partial\varphi}{\partial\tau}\right)^2\right]+\frac{\partial\varphi}{\partial\xi}\frac{\partial}{\partial\tau}\left(s^2\frac{\partial\varphi}{\partial\tau}\right)+\frac{\partial p}{\partial\xi}=0, \tag{1.3}$$

$$\frac{\partial s}{\partial\theta}\left[\frac{\partial^2 s}{\partial\tau^2}-s\left(\frac{\partial\varphi}{\partial\tau}\right)^2\right]+\frac{\partial\varphi}{\partial\theta}\frac{\partial}{\partial\tau}\left(s^2\frac{\partial\varphi}{\partial\tau}\right)+\frac{\partial p}{\partial\theta}=0,$$

$$\frac{\partial s}{\partial\xi}\frac{\partial\varphi}{\partial\theta}-\frac{\partial s}{\partial\theta}\frac{\partial\varphi}{\partial\xi}=\frac{\xi+1}{s}.$$

In these equations the required functions s, φ, and p depend on the variables ξ, θ, and τ. For simplicity the dimensionless and dimensional pressures in Eqs. (1.3) are denoted by the same symbol. The definition of the Lagrangian variables and Eqs. (1.2) yield the following initial conditions for the functions $s=s(\xi, \theta, \tau)$ and $\varphi=\varphi(\xi, \theta, \tau)$:

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$$s = \xi + 1, \ \varphi = \theta \ \text{as} \ \tau = 0. \tag{1.4}$$

Plane axisymmetric motions correspond to the solutions of Eqs. (1.3) in which

$$s = \sigma(\xi, \tau), \ \varphi = \theta + \gamma(\xi, \tau), \ p = p_0(\xi, \tau). \tag{1.5}$$

The integration of Eqs. (1.3) under the assumptions (1.5) leads to the following expressions:

$$\sigma = [(\xi + 1)^2 - \Phi(\tau)]^{1/2}, \ \partial\gamma/\partial\tau = (\xi + 1)^2\omega(\xi) \times [(\xi + 1)^2 - \Phi(\tau)]^{-1}, \tag{1.6}$$

$$\frac{\partial p_0}{\partial \xi} = (\xi + 1)\,\sigma^{-4}\left[\omega^2(\xi)(\xi + 1)^4 + \frac{1}{2}\sigma^2\frac{d^2\Phi}{d\tau^2} + \frac{1}{4}\left(\frac{d\Phi}{d\tau}\right)^2\right].$$

In Eqs. (1.6) the function $\omega(\xi)$ gives the initial distribution of the angular velocity, and $\Phi(\tau)$ determines the law of motion of the ring.

By satisfying the boundary conditions at the free surfaces of the ring $\xi = 0$ and $\xi = l = (R_{20} - R_{10})/R_{10}$ an ordinary second order differential equation is obtained for $\Phi(\tau)$. The initial conditions for this function are determined by conditions (1.4) and the specification of the radial component of velocity at the time $\tau = 0$.

2. Equations of Small Perturbations. The behavior of small perturbations for motions of an ideal incompressible liquid described by Eqs. (1.6) will be investigated within the framework of the linear theory. Suppose the perturbed motion is described by the expressions

$$s = \sigma(\xi, \tau) + \sum_{n=-\infty}^{+\infty} R_n(\xi, \tau)\,e^{in\theta}, \tag{2.1}$$

$$\varphi = \theta + \gamma(\xi, \tau) + \sum_{n=-\infty}^{+\infty} \Lambda_n(\xi, \tau)\,e^{in\theta}, \quad p = p_0(\xi, \tau) + \sum_{n=-\infty}^{+\infty} \Pi_n(\xi, \tau)\,e^{in\theta},$$

where the amplitudes of the perturbations R_n, Λ_n, and Π_n and all their derivatives are assumed sufficiently small.

Substituting Eqs. (2.1) into (1.3) and discarding terms of higher order of smallness in comparison with the amplitudes of the perturbations leads to the following system of equations:

$$\frac{\partial\sigma}{\partial\xi}\left[\frac{\partial^2 R_n}{\partial\tau^2} - \left(\frac{\partial\gamma}{\partial\tau}\right)^2 R_n\right] - \left(\frac{\partial\sigma}{\partial\xi}\right)^{-1}\frac{\partial p_0}{\partial\xi}\left(\frac{\partial R_n}{\partial\xi} - in\frac{\partial\gamma}{\partial\xi}R_n\right) - 2(\xi + 1)\frac{\partial\gamma}{\partial\tau}\frac{\partial\Lambda_n}{\partial\tau} + \frac{\partial\Pi_n}{\partial\xi} - in\frac{\partial\gamma}{\partial\xi}\Pi_n = 0, \tag{2.2}$$

$$\frac{\partial}{\partial\tau}\left(\sigma^2\frac{\partial\Lambda_n}{\partial\tau}\right) + 2\sigma\frac{\partial\gamma}{\partial\tau}\frac{\partial R_n}{\partial\tau} - 2\frac{\partial\sigma}{\partial\tau}\frac{\partial\gamma}{\partial\tau}R_n - in\left(\frac{\partial\sigma}{\partial\xi}\right)^{-1}\frac{\partial p_0}{\partial\xi}R_n + in\,\Pi_n = 0,$$

$$\frac{\partial}{\partial\xi}(\sigma R_n) - in\frac{\partial\gamma}{\partial\xi}\sigma R_n + in(\xi + 1)\Lambda_n = 0.$$

It follows from the form of Eqs. (2.2) that for $n \neq 0$ they can be reduced to a single fourth order equation for $R_n(\xi, \tau)$. After determining this function the amplitudes Λ_n and Π_n can be found from the second and third of Eqs. (2.2).

The fourth order equation obtained is generally of rather unwieldy form. Since the coefficients in this equation depend on both ξ and τ, it is practically impossible to find the asymptotic solution of the corresponding initial-boundary value problem without further simplifications. Investigation of the properties of this fourth order equation enabled us to establish the following fact: If the initial angular velocity $\omega(\xi)$ has the form

$$\omega(\xi) = \omega_0 + \omega_1/(\xi + 1)^2, \tag{2.3}$$

the equation for R_n can be written in the compact form

$$D_1 D_2 D_3 D_4(\sigma R_n) = 0. \tag{2.4}$$

In (2.3) the quantities ω_0 and ω_1 are arbitrary constants. The initial distribution (2.3) corresponds to the superposition of the potential vorticity and rotation of the liquid as a solid. In Eq. (2.4) the D_i $(i = 1, \ldots, 4)$ denote the following differential operators:

$$D_1 = \frac{\partial}{\partial\tau}, \quad D_2 = e^{in\gamma}\sigma^{-n}\frac{\partial}{\partial\xi}, \quad D_3 = (\xi + 1)^{-1}\sigma^{2(n+1)}\frac{\partial}{\partial\xi}, \quad D_4 = e^{-in\gamma}\sigma^{-n}\frac{\partial}{\partial\tau}.$$

Equation (2.4) is easily integrated and yields the expression

$$\frac{\partial}{\partial\tau}(\sigma R_n) = e^{in\gamma}\sigma^n\int_0^\xi e^{-in\gamma(\zeta,\tau)}\sigma^{-n}(\zeta, \tau)\,A_n(\zeta)\,d\zeta - e^{in\gamma}\sigma^{-n}\int_0^\xi e^{-in\gamma(\zeta,\tau)}\,\sigma^n(\zeta, \tau)\,A_n(\zeta)\,d\zeta + e^{in\gamma}\left(\sigma^n u_n(\tau) + \sigma^{-n}v_n(\tau)\right), \tag{2.5}$$

where $A_n(\xi)$, $u_n(\tau)$, and $v_n(\tau)$ are arbitrary functions to be determined from the initial and boundary conditions for the perturbed motion. By specifying the initial perturbation

$$\partial R_n/\partial\tau = V_n(\xi) \quad \text{at} \quad \tau = 0$$

of the radial component of velocity the function $A_n(\xi)$ is determined at once:

$$A_n(\xi) = \frac{1}{2n}\left[(\xi+1)^2\frac{d^2V_n}{d\xi^2} + 3(\xi+1)\frac{dV_n}{d\xi} - (n^2-1)V_n\right].$$

It can be established that the boundary conditions at free surfaces in perturbed motion are reduced to a system of two ordinary second order differential equations for $u_n(\tau)$ and $v_n(\tau)$.

Thus, it has been established that for solutions of the hydrodynamics equations given by (1.6) and (2.3) the analysis of small perturbations in the class of plane motions is reduced to an investigation of the Cauchy problem for ordinary differential equations.

3. Motion toward the Center of a Rotating Ring. We apply the result obtained to the investigation of small perturbations of the motion of a rotating ring toward the center under the action of a variable external pressure. The problem of the motion of a rotating ring toward the center is considered in the following arrangement.

Suppose at times $t \le 0$ a ring $R_{10} \le r \le R_{20}$ of an ideal incompressible liquid rotating as a rigid body with an angular velocity Ω_0 is in equilibrium under the action of a pressure drop and capillary forces with surface tensions $\beta_0^!$ and $\beta_l^!$ respectively on the inner and outer surfaces of the ring. Different surface tensions on the inner and outer surfaces occur when the media inside and outside the ring have different physical properties. For $t > 0$ a pressure $p = P(t)$ acts on the outer surface of the ring, and the inner surfaces of the ring is free. It is required to determine the resulting motion of the ring.

The solution of the problem posed is given in dimensionless variables by Eqs. (1.6), where $\omega(\xi) \equiv \Omega_0 T = \omega_0$ is the constant initial angular velocity and $\Phi(\tau)$ is the solution of the Cauchy problem.

$$\left(\omega_0^2\Phi + \frac{1}{4}\frac{d^2\Phi}{d\tau^2}\right)\ln\frac{\sigma_l^2}{\sigma_0^2} - \frac{1}{2}\left(\sigma_l^{-2} - \sigma_0^{-2}\right)\left[\omega_0^2\Phi^2 + \frac{1}{4}\left(\frac{d\Phi}{d\tau}\right)^2\right] = P(\tau) - \frac{1}{2}\omega_0^2(l^2+2l) + \beta_l\sigma_l^{-1} + \beta_0\sigma_0^{-1}, \quad \Phi(0) = \frac{d\Phi}{d\tau}(0) = 0, \tag{3.1}$$

where $\beta_0 = T^2(\rho_0 R_{10}^3)^{-1}\beta_0^!$ and $\beta_l = T^2(\rho_0 R_{10}^3)^{-1}\beta_l^!$ are the dimensionless surface tensions; $\sigma_0(\tau) \equiv [1 - \Phi(\tau)]^{1/2}$ and $\sigma_l(\tau) \equiv [(l+1)^2 - \Phi(\tau)]^{1/2}$ are functions giving the laws of motion of the inner and outer surfaces of the ring, respectively.

A simple analysis of problem (3.1) shows that two qualitatively different kinds of motion of the ring are possible depending on the sign of the quantity

$$\varkappa = P(0) - \frac{1}{2}\omega_0^2(l^2+2l) + \beta_0 + \frac{\beta_l}{l+1}.$$

For $\varkappa > 0$ the ring begins to be compressed, and for $\varkappa < 0$ to expand. Further, if $\omega_0 \ne 0$ and for all $t > 0$ the function P(t) satisfies the inequalities $0 < P_0 \le P(t) \le P_1 < \infty$ the inner radius of the ring $R_1(t)$ will vary over finite limits $0 < R_* \le R_1(t) \le R_{**} < \infty$ and the possible regimes when the critical values are R_* or R_{**} will be reached after an infinite time. If the pressure at the outer surface of the rotating ring is constant ($p = P_0 > 0$) the ring will pulsate with an amplitude and period which are constant in time.

Henceforth we restrict ourselves to the case of the convergence of a ring toward the center under the assumption that $R_*/R_{10} \ll 1$. The latter condition can always be satisfied, since for $\omega_0 = 0$ the critical radius $R_* = 0$.

We assume that this motion is perturbed either by an initial distortion of the shape of the boundaries of the ring from circular form or by the asymmetry of the impulse. Suppose that in the perturbed motion the inner surface of the ring is given by the equation

$$\xi = \nu(\theta) = \sum_{n=-\infty}^{+\infty} \nu_n e^{in\theta},$$

and the outer surface of the ring by the equation

$$\xi = l + \mu(\theta) = l + \sum_{n=-\infty}^{+\infty} \mu_n e^{in\theta}.$$

The amplitudes ν_n and μ_n of the initial perturbations of the boundaries of the ring are assumed sufficiently small. We assume also that perturbations are introduced into the distribution of initial velocities, i.e.

$$\partial R_n/\partial\tau = V_n(\xi) \quad \text{at} \quad \tau = 0,$$

and into the law of action of the pressure at the outer surface of the ring

$$p = P(\tau) + \sum_{n=-\infty}^{+\infty} \pi_n(\tau)\, e^{in\theta} \quad \text{at} \quad \xi = l + \mu(\theta).$$

The radial deviations of the free surfaces in the perturbed motion from the free surfaces in the initial motion very clearly characterize the effect of small perturbations on the motion of the ring. We denote by $H_0(\theta, \tau)$ and $H_l(\theta, \tau)$ the values of the deviations for the inner and outer surfaces of the ring respectively. In the linear approximation these quantities are given by the equations

$$H_0(\theta, \tau) = \sigma(\nu(\theta), \tau) + R(\nu(\theta), \theta, \tau) - \sigma_0(\tau) \cong \sigma_0^{-1}(\tau) \sum_{n=-\infty}^{+\infty} [\nu_n + \sigma_0(\tau) R_n(0, \tau)]\, e^{in\theta} = \sum_{n=-\infty}^{+\infty} H_{0n}(\tau)\, e^{in\theta}, \tag{3.2}$$

$$H_l(\theta, \tau) = \sigma(l + \mu(\theta), \tau) + R(l + \mu(\theta), \theta, \tau) - \sigma_l(\tau) \cong \sigma_l^{-1}(\tau) \sum_{n=-\infty}^{+\infty} [(l+1)\mu_n + \sigma_l(\tau) R_n(l, \tau)]\, e^{in\theta} = \sum_{n=-\infty}^{+\infty} H_{ln}(\tau)\, e^{in\theta}.$$

The equations which must be satisfied by the functions $H_{0n}(\tau)$ and $H_{ln}(\tau)$ of Eqs. (3.2) follow from the boundary conditions at the free surfaces of the ring in perturbed motion and from the obvious relations of these functions to the functions $u_n(\tau)$ and $v_n(\tau)$ given by Eq. (2.5). This system of equations is too cumbersome to present here, but we note that in general it consists of two inhomogeneous second order ordinary differential equations. The coefficients in this system of equations are regular everywhere except at the point $\tau = \tau_*$ determined by the equation $\sigma_0(\tau_*) = 0$. At $\tau = \tau_*$ some of the coefficients have singularities of the form $\sigma_0^{-k}(-\ln \sigma_0)^{\lambda}$, where k and λ are positive rational numbers. We note that the equations of small irrotational perturbations of the inertial motion of a ring have similar singularities [1]. The presence of a logarithmic singularity in the equations of small perturbations complicates finding the asymptotic representation of the solution of the Cauchy problem in the neighborhood of the singular point $\tau = \tau_*$, since there is no general theory for constructing asymptotic expansions when the singular points are not poles [6].

Since a detailed discussion of the asymptotic analysis of the solutions of the system of equations under discussion in the neighborhood of the time of collapse of the ring $\tau = \tau^*$ cannot be given in this paper, we present the results of this analysis.

At all times $\tau \in [0, \tau_*]$ during the compression of the ring the amplitudes $H_{ln}(\tau)$ of the perturbations of the outer surface of the ring remain bounded. The surface tension exerts a stabilizing effect on the development of perturbations; i.e., as $n \to \infty$ the amplitudes $H_{0n}(\tau)$ and $H_{ln}(\tau)$ are bounded functions in the interval $[0, \tau_*]$. For all the types of perturbations enumerated above the behavior of the amplitudes $H_{0n}(\tau)$ of the perturbations of the inner surface of a ring as $\tau \to \tau_*$ differs in the following cases: a) $\omega_0 \neq 0$; b) $\omega_0 = 0$, $n = 1$; c) $\omega_0 = 0$, $n > 1$. It should be noted that for $\omega_0 \neq 0$ the condition $R_*/R_{10} \ll 1$ justifies the use of the asymptotic analysis to investigate the solutions of the equations of small perturbations in the neighborhood of the time of convergence of the ring to the critical position characterized by the equation $R_1 = R_*$.

A study of the properties of the solutions of the equations of small perturbations for cases a)-c) leads respectively to the following asymptotic representations of the function $H_{0n}(\tau)$:

$$H_{0n} = L_n(\tau) \exp\left[\frac{4}{3} i \sqrt{n} \frac{\omega_0}{Q} (-\ln \sigma_0)^{3/2} (1 + o(1))\right]; \tag{3.3}$$

$$H_{01} = M_1(\tau) + N_1(\tau)(-\ln \sigma_0)^{3/2}(1 + o(1));$$

$$H_{0n} = M_n(\tau)(-\ln \sigma_0)^{1/4} \exp(i\sqrt{n-1} \ln \sigma_0)(1 + o(1)).$$

In these expressions $L_n(\tau)$, $N_1(\tau)$, and $M_n(\tau)$ are functions which are bounded in the interval $[0, \tau_*]$, and the constant Q is

$$Q = 2\left[\int_0^{\tau_*} P(\tau)\, \Phi'(\tau)\, d\tau\right]^{1/2}.$$

It follows from (3.3) that rotation plays a double role in determining the behavior of small perturbations during the collapse of a ring. On the one hand it decreases the growth of the amplitudes of the perturbations, and on the other hand it increases their oscillation. The stabilizing effect of the rotation of a liquid on the development of small perturbations was noted in [7] also, where the stability of stationary plane parallel potential vorticity with respect to plane irrotational perturbations was studied.

In conclusion we note that the effects of rotational perturbations on the stability of contraction of a ring of ideal incompressible liquids are indirect. A similar asymptotic analysis for the equations of irrotational perturbations of the inertial motion of a ring presented in [1] shows that the asymptotic form of the radial deflections of the inner surface of the ring will coincide with the asymptotic behavior of the functions $H_{0n}(\tau)$ in cases b) and c). Consequently in the collapse of a ring of ideal incompressible liquid perturbations of general form develop similarly to irrotational perturbations.

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APPLICATION OF EXACT SOLUTIONS OF THE "SHALLOW WATER" EQUATIONS TO THE EXPLANATION OF THE SIMPLEST FLOWS

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Stationary solutions of the differential equations of the theory of "shallow water" with axial symmetry are given by the implications of these equations:

$$rhu = Q = Q_0/2\pi\rho = \text{const}; \tag{1}$$

$$rv = D = \text{const}; \tag{2}$$

$$\frac{u^2+v^2}{2} + gh = C = \text{const}, \tag{3}$$

where $h = h(r)$ is the height of an incompressible fluid layer of density ρ, $u = u(r)$, $v = v(r)$ are, respectively, the radial and circumferential components of the fluid velocity vector which is considered constant along the vertical in the whole layer $0 \leq z \leq h(r)$ in the "shallow water" approximation, g is the acceleration of gravity, and Q_0 is the fluid discharge through any section $r = \text{const} \geq r_0$.

From (1)-(3) we have the relationship

$$\frac{Q^2}{2r^2} = \frac{h^2(C-gh)}{1+\varkappa^2 h^2} = \varphi(h, C, \varkappa),\ \varkappa = D/Q, \tag{4}$$

which implicitly defines the dependence $h = h(r)$ for known Q, C, D.

Graphs of the function $\varphi(h, C, \varkappa)$ are presented in Fig. 1 for $\varkappa = 0$ and $\varkappa = 1$.

The method of the graphical determination of the dependence $h = h(r)$ is evident from (4), and it is also clear from Fig. 1 that a stationary axisymmetric solution of the "shallow water" equations exists only for

$$r > r_*(Q, C, \varkappa) > r_*(Q, C, 0) = (\sqrt{27/8})gQC^{-3/2}.$$

Hence (4) yields two solutions corresponding to two different fluid flow regimes.

The first flow regime corresponds to the dependence $h = h(r)$, determined from (4) for $0 < h < h_*(C, \varkappa)$, and the second for $h_*(C, \varkappa) < h < C/g$. Here $h_*(C, \varkappa)$ denotes the value of h for which $\varphi(h, C, \varkappa)$ reaches the maximum. It can be seen that for $\varkappa \neq 0$

$$h_*(C, \varkappa) < h(C, 0) = 2C/3g.$$

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